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Gravitational wave background from Population III black hole formation

José C. N. de Araujo⋆, Oswaldo D. Miranda† and Odylio D. Aguiar‡
Instituto Nacional de Pesquisas Espaciais - Divisão de Astrofísica
Av. dos Astronautas 1758, São José dos Campos, 12227-010 SP, Brazil

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ABSTRACT

We study the generation of a stochastic gravitational wave (GW) background produced from a population of core-collapse supernovae, which form black holes in scenarios of structure formation. We obtain, for example, that the formation of a population (Population III) of black holes, in cold dark matter scenarios, could generate a stochastic GW background with a maximum amplitude of $h_{\rm BG} \simeq 10^{-24}$ and corresponding closure energy density of $\Omega_{\rm GW} \sim 10^{-7}$, in the frequency band $\nu_{\rm obs} \simeq 30 - 470\,{\rm Hz}$ (assuming a maximum efficiency of generation of GWs, namely, $\varepsilon_{\rm GW_{max}} = 7 \times 10^{-4}$) for stars forming at redshifts $z \simeq 30 - 10$. We show that it will be possible in the future to detect this isotropic GW background by correlating signals of a pair of 'advanced' LIGO observatories (LIGO III) at a signal-to-noise ratio of $\simeq 40$. We discuss what astrophysical information could be obtained from a positive (or even a negative) detection of such a GW background generated in scenarios such as those studied here. One of them is the possibility of obtaining the initial and final redshifts of the emission period from the observed spectrum of GWs.

Key words: black hole physics — gravitation — cosmology:theory.

1 INTRODUCTION

The detection of gravitational waves (GWs) will open up a new era in the history of astronomy and transform research in general relativity into an observational/theoretical study (Schutz 1999).

The detection of GWs will directly verify the predictions of general relativity theory concerning the existence or not of such waves, as well as other theories of gravity (Thorne 1987). The information provided by such waves is completely different when compared to that provided by electromagnetic waves. GWs carry detailed information on the coherent bulk motions of matter, such as in collapsing stellar cores or coherent vibrations of space-time curvature as produced, for example, by black holes. On the other hand, electromagnetic waves are usually an incoherent superposition of emissions from individual atoms, molecules and charged particles.

There is a host of possible astrophysical sources of GWs: namely, supernovae, the collapse of a star or star cluster to form a black hole, inspiral and coalescence of compact binaries, the fall of stars and black holes into supermassive black holes, rotating neutron stars, ordinary binary stars, relics of the big bang, vibrating or colliding of monopoles, cosmic strings and cosmic bubbles, among others (see, e.g., Thorne 1987, 1996, 1997; Schutz 1996, 1999).

From the theoretical point of view there has been a great effort to study which are the most promising sources of GWs to be detected. In particular, the waveforms, the characteristic frequencies and the number of sources per year that one expects to observe are questions that have been addressed (see Thorne 1997; Schultz 1999; Grishchuk et al. 2000 for a review). In a few years, starting from the observations (waveforms, amplitudes, polarizations, etc.), it will be possible really to understand how GW emission is generated by the astrophysical sources.

Because of the fact that GWs are produced by a large variety of astrophysical sources and cosmological phenomena, it is quite probable that the Universe is pervaded by a background of such waves. A variety of binary stars (ordinary, compact or combinations of them), Population III stars, phase transitions in the early Universe and cosmic strings are examples of sources that could produce such putative GW background (Thorne 1987).

As GWs possess a very weak interaction with matter, passing through it without being disturbed, once detected they can provide information on the physical conditions from

⋆ jcarlos@das.inpe.br
† Present address: Department of Physics, Washington University, Campus Box 1105, One Brookings Drive, St. Louis - MO 63130-4899 - USA; oswaldo@hbar.wustl.edu
‡ odylio@das.inpe.br

the era in which they were produced. In principle, it will be possible to get information from the epoch when the galaxies and stars started to form and evolve.

Concerning the production of GW backgrounds, it is worth mentioning that recently Blair & Ju (1996) and Ferrari, Matarrese & Schneider (1999a,b) studied the cosmological GW background produced by supernovae explosions that took place in the redshift range $0 < z < 5$.

On the other hand, from considerations based on the Gunn-Peterson effect (Gunn & Peterson 1965), it is widely accepted that the Universe underwent a reheating (or reionization) phase between the standard recombination epoch (at $z \sim 1000$) and $z > 5$ (see Haiman & Loeb 1997; Loeb & Barkana 2001 for a review). However, at what redshift the reionization occurred is still an open question (Loeb & Barkana 2001), although recent studies conclude that it occurred at redshifts in the range $6 < z < 30$ (Venkatesan 2000; Tegmark & Zaldarriaga 2000; Schmalzing et al. 2000; Loeb & Barkana 2001). It is worth noting that present and future cosmic background radiation (CBR) studies can impose some constraints on the reionization phase of the Universe (see, e.g., Tegmark & Zaldarriaga 2000; Loeb & Barkana 2001).

Although different models could possibly explain the reionization of the Universe, it is widely accepted that most of the contribution to the reionization is related to the formation and evolution of pre-galactic objects (sometimes called Population III objects) at high redshift ($z > 10$), such as subgalactic objects ($M < 10^9 \mathrm{M}_\odot$) and stars formed from them (Loeb & Barkana 2001). This putative epoch where the formation of the Population III objects took place, and where the consequent reionization and reheating of the Universe occurred, marked the end of an epoch named 'dark age' (see, e.g., Rees 1998). Also, note that the metalicity of $\sim 10^{-2} Z_\odot$ found in high $-z$ Lyα forest clouds (Songaila & Cowie 1996; Ellison et al. 2000) is consistent with a stellar population formed at $z > 5$ (Venkatesan 2000).

The history of the Universe during the formation of the Population III objects could be investigated in the future with the *New Generation of Space Telescope* (*NGST*; Rees 1998) and also, in principle, with GW observatories. Besides the reheating and reionization phases, putative Population III stars could produce GWs, particularly, from the formation of neutron stars and black holes. Also, after the dark age, the epoch of the first light could be studied with large radio telescopes such as the Giant Meter Wavelength Telescope (GMRT) or the Square Kilometer Array Radio Telescope (SQA; Meiksin 1999).

When the high-mass stars died as supernovae, they left stellar black holes as remnants. The formation of these stellar Population III black holes can, in principle, produce a GW background detectable by GW observatories. It is worth mentioning that a significant amount of GWs can also be produced during the formation of neutron stars. However, because this depends on the equation of state for the neutron star, which is not well defined, we consider here only the contribution of the black holes. Another possibility would be the generation of GWs through the so-called r-mode instability (Anderson 1998), which should be important for young, hot and rapidly rotating neutron stars, but we leave this issue for another study, to appear elsewhere.

In the present study we have adopted a stellar generation with a Salpeter initial mass function (IMF) as well as different stellar formation epochs. We then discuss what conclusions would be drawn whether (or not) the stochastic background studied here is detected by the forthcoming GW observatories such as LIGO and VIRGO.

The paper is organized as follows. In Section 2 we present the basic ideas on the collapse of the first clouds and on the resulting stellar formation; in Section 3 we describe how to calculate the GW background produced during the formation of black holes in this scenario; in Section 4 we present and discuss the numerical results; in Section 5 we consider the detectability of the putative GW background produced by the Population III black holes; and finally in Section 6 we present the conclusions.

2 POPULATION III OBJECTS AND THE FIRST STARS

The current theory of structure formation, based on cold dark matter (CDM) models, predicts that the first objects to collapse, the so-called Population III objects or mini-haloes, had a total mass of $\sim 10^6 M_\odot$ and a formation epoch $z \sim 10-50$ (see Tegmark et al. 1997, and references therein). The first stars, the Population III stars, started forming in these Population III objects of $\sim 10^6 M_\odot$ and subsequently in more massive mini-haloes (see Haiman & Loeb 1997; Venkatesan 2000; Loeb & Barkana 2001).

As a result of the formation of the first stars, a population of stellar black holes is formed after the supernova explosions associated with the high-mass stars. Then, knowing the law of distribution of stellar masses, it is possible to obtain the number of stars that explode as supernovae, and so it is possible to determine the number (or event rate) of stellar black holes left as remnants.

Thus, to proceed, the distribution function of stellar masses, the stellar IMF, for the first stars is required. Here the Salpeter IMF is adopted, namely

$$\phi(m) = A m^{-(1+x)}, \qquad (1)$$

where A is the normalization constant and $x = 1.35$ (our fiducial value). The normalization of the IMF is obtained through the relation

$$\int_{m_\mathrm{l}}^{m_\mathrm{u}} m\phi(m)dm = 1, \qquad (2)$$

where we consider $m_\mathrm{l} = 0.1\,\mathrm{M}_\odot$ and $m_\mathrm{u} = 125\,\mathrm{M}_\odot$. It is worth noting that some authors argue (see, e.g., Gilmore 2001) that there is evidence supporting the universality of the IMF, even for the first stars; on the other hand, other authors (see, e.g., Scalo 1998, among others) argue that the IMF may not be universal. In particular, the universality of the Salpeter exponent ($x = 1.35$) has been studied by recent evolutionary models for the Magellanic Clouds (de Freitas-Pacheco 1998). Some models, particularly those of the Large Magellanic Cloud, take into account constraints on the star formation history imposed by recent data on color-magnitude diagrams of field star clouds, showing that a steeper exponent, $x = 2.0$ is necessary to resolve the excessive production of iron obtained if one takes into account the Salpeter law ($x = 1.35$). Furthermore, concerning the star formation at high redshift, the IMF could be biased toward

high-mass stars, when compared to the solar neighborhood IMF, as a result of the absence of metals (Bromm, Coppi & Larson 1999, 2001).

Then, for the standard IMF, the mass fraction of black holes produced as remnants of the stellar evolution is

$$f_{\rm BH} = \int_{\rm m_{min}}^{m_{\rm u}} M_{\rm r}\phi(m)dm, \tag{3}$$

where $m_{\rm min}$ is the minimum stellar mass capable of producing a black hole at the end of its life, and $M_{\rm r}$ is the mass of the remnant black hole. Timmes, Woosley & Wheaver (1995) (see also Woosley & Timmes 1996) obtain, from stellar evolution calculations, that the minimal progenitor mass to form black holes is $18 \leq m_{\rm min}/{\rm M}_{\odot} \leq 30$ depending on the stellar iron core mass. Thus, we assume that the minimum mass capable of forming a remnant black hole is $m_{\rm min} = 25\,{\rm M}_{\odot}$. For the remnant, $M_{\rm r}$, we take $M_{\rm r} = \alpha\, m$, where m is the mass of the progenitor star and $\alpha = 0.1$ (see, e.g., Ferrari et al. 1999a, b). With these considerations at hand, the mass fraction of black holes reads $f_{\rm BH} = 6.8 \times 10^{-2} \times \alpha \simeq 6.8 \times 10^{-3}$ for $x = 1.35$.

To assess the role of possible IMF variations in our results, other values of x have also been considered. Besides the standard IMF, two others have been studied, namely, with $x = 0.3$ and $x = 1.85$, which yield ten times and one-tenth of the mass fraction of black holes of the standard IMF, respectively.

It is worth mentioning that stars formed with masses greater than $8\,{\rm M}_{\odot}$ to $\sim\ 25\,{\rm M}_{\odot}$ also finish their lives as supernovae. Numerical studies have shown that these stars leave neutron star remnants, after forming iron cores with masses near the Chandrasekhar limit (Woosley & Timmes 1996). These stars are important for injecting energy into the ambient medium and regulating the feedback of stellar formation. In the present paper only the generation of GWs that come from black holes formation have been studied, and so the progenitors of interest are stars with masses in the interval $25 \leq m/{\rm M}_{\odot} \leq 125$.

3 GRAVITATIONAL WAVE PRODUCTION

The GWs can be characterized by their dimensionless amplitude, h, and frequency, ν. The spectral energy density, the flux of GWs, received on Earth, F_ν, in erg cm^{-2}s^{-1}Hz^{-1}, is (see, e.g., Douglass & Braginsky 1979; Hils, Bender & Webbink 1990)

$$F_\nu = \frac{c^3 s_{\rm h}\omega_{\rm obs}^2}{16\pi G}, \tag{4}$$

where $\omega_{\rm obs} = 2\pi\nu_{\rm obs}$, with $\nu_{\rm obs}$ the GW frequency (Hz) observed on Earth, c is the velocity of light, G is the gravitational constant and $\sqrt{s_{\rm h}}$ is the strain amplitude of the GW (Hz$^{-1/2}$).

The stochastic GW background produced by gravitational collapses that lead to black holes would have a spectral density of the flux of GWs and strain amplitude also related to the above equation (4). Therefore, in the above equation the strain amplitude takes into account the star formation history occurring at the 'first light', just after the 'dark age' epoch. The strain amplitude at a given frequency, at the present time, is a contribution of black holes with different masses at different redshifts. Thus, the ensemble of black holes formed produces a background whose characteristic strain amplitude at the present time is $\sqrt{s_{\rm h}}$.

On the other hand, the spectral density of the flux can be written as (Ferrari et al. 1999a,b)

$$F_\nu = \int_{z_{\rm cf}}^{z_{\rm ci}}\int_{m_{\rm min}}^{m_{\rm u}} f_\nu(\nu_{\rm obs})dR_{\rm BH}(m,z), \tag{5}$$

where $f_\nu(\nu_{\rm obs})$ is the energy flux per unit of frequency (in erg cm^{-2}Hz^{-1}) produced by the formation of a unique black hole and $dR_{\rm BH}$ is the differential rate of black holes formation.

The above equation takes into account the contribution of different masses that collapse to form black holes occurring between redshifts $z_{\rm ci}$ and $z_{\rm cf}$ (beginning and end of the star formation phase, respectively) that produce a signal at the same frequency $\nu_{\rm obs}$. On the other hand, we can write $f_\nu(\nu_{\rm obs})$ (Carr 1980) as

$$f_\nu(\nu_{\rm obs}) = \frac{\pi c^3}{2G}h_{\rm BH}^2, \tag{6}$$

where $h_{\rm BH}$ is the dimensionless amplitude produced by the collapse to a black hole of a given star with mass m that generates at the present time a signal with frequency $\nu_{\rm obs}$. Then, the resulting equation for the spectral density of the flux is

$$F_\nu = \frac{\pi c^3}{2G}\int h_{\rm BH}^2 dR. \tag{7}$$

From the above equations we obtain for the strain amplitude

$$s_{\rm h} = \frac{1}{\nu_{\rm obs}^2}\int h_{\rm BH}^2 dR. \tag{8}$$

Thus, the dimensionless amplitude reads

$$h_{\rm BG}^2 = \frac{1}{\nu_{\rm obs}}\int h_{\rm BH}^2 dR, \tag{9}$$

(see de Araujo, Miranda & Aguiar 2000 for details).

The dimensionless amplitude produced by the collapse of a star, or star cluster, to form a black hole is (Thorne 1987)

$$h_{\rm BH} = \left(\frac{15}{2\pi}\varepsilon_{\rm GW}\right)^{1/2}\frac{G}{c^2}\frac{M_{\rm r}}{r_{\rm z}} \simeq 7.4 \times 10^{-20}\varepsilon_{\rm GW}^{1/2}\left(\frac{M_{\rm r}}{\mathbf{M}_{\odot}}\right)\left(\frac{r_{\rm z}}{1{\rm Mpc}}\right)^{-1}, \tag{10}$$

where $\varepsilon_{\rm GW}$ is the efficiency of generation of GWs and $r_{\rm z}$ is the distance to the source.

The collapse of a star to a black hole produces a signal with frequency (Thorne 1987)

$$\nu_{\rm obs} = \frac{1}{5\pi M_{\rm r}}\frac{c^3}{G}(1+z)^{-1} \simeq 1.3 \times 10^4{\rm Hz}\left(\frac{{\rm M}_{\odot}}{M_{\rm r}}\right)(1+z)^{-1}, \tag{11}$$

where the factor $(1+z)^{-1}$ takes into account the redshift effect on the emission frequency, that is, a signal emitted at

frequency $\nu_{\rm e}$ at redshift z is observed at frequency $\nu_{\rm obs} = \nu_{\rm e}(1+z)^{-1}$. The observed signal is in the range

$$\frac{1.04\times 10^3}{(1+z_{\rm ci})}\ {\rm Hz} \le \nu_{\rm obs} \le \frac{5.2\times 10^3}{(1+z_{\rm cf})}\ {\rm Hz}, \tag{12}$$

obtained using the mass upper limit $m_{\rm u} = 125\,{\rm M}_\odot$, the mass lower limit $m_{\rm min} = 25\,{\rm M}_\odot$, and $\alpha = 0.1$.

For the differential rate of black hole formation we have

$$dR_{\rm BH} = \dot\rho_\star(z)\frac{dV}{dz}\phi(m)dmdz, \tag{13}$$

where $\dot\rho_\star(z)$ is the star formation rate (SFR) density (in ${\rm M}_\odot\,{\rm yr}^{-1}\,{\rm Mpc}^{-3}$) and dV is the comoving volume element.

From the above equations we obtain for the dimensionless amplitude

$$\begin{aligned} h^2_{\rm BG} \;=\; & \frac{(7.4\times 10^{-20}\alpha)^2\varepsilon_{\rm GW}}{\nu_{\rm obs}} \\ & \times\left[\int_{z_{\rm cf}}^{z_{\rm ci}}\int_{m_{\rm min}}^{m_{\rm u}}\left(\frac{m}{{\rm M}_\odot}\right)^2\left(\frac{d_{\rm L}}{1{\rm Mpc}}\right)^{-2}\dot\rho_\star(z)\right. \\ & \left.\times\frac{dV}{dz}\phi(m)dmdz\right]. \end{aligned} \tag{14}$$

In equation (14) $d_{\rm L}$ is the luminosity distance to the source. The comoving volume element is given by

$$dV = 4\pi\left(\frac{c}{H_0}\right)r_{\rm z}^2\mathcal{F}(\Omega_{\rm M},\Omega_\Lambda,z)dz, \tag{15}$$

with

$$\mathcal{F}(\Omega_{\rm M},\Omega_\Lambda,z) \equiv \frac{1}{\sqrt{(1+z)^2(1+\Omega_{\rm M}z)-z(2+z)\Omega_\Lambda}}, \tag{16}$$

and the comoving distance, $r_{\rm z}$, is

$$r_{\rm z} = \frac{c}{H_0\sqrt{|\Omega_{\rm k}|}}S\left(\sqrt{|\Omega_{\rm k}|}\int_0^z\frac{dz'}{\mathcal{F}(\Omega_{\rm M},\Omega_\Lambda,z')}\right), \tag{17}$$

where

$$\Omega_{\rm M} = \Omega_{\rm DM} + \Omega_{\rm B} \qquad \text{and} \qquad 1 = \Omega_{\rm k} + \Omega_{\rm M} + \Omega_\Lambda \tag{18}$$

are the usual density parameters for the matter (M), i.e., dark matter (DM) plus baryonic matter (B), curvature (k) and cosmological constant (Λ). The function S is given by

$$S(x) = \begin{cases}\sin x & \text{if closed,}\\ x & \text{if flat,}\\ \sinh x & \text{if open.}\end{cases} \tag{19}$$

The comoving distance is related to the luminosity distance by

$$d_{\rm L} = r_{\rm z}(1+z). \tag{20}$$

The set of equations presented above can be used to find the dimensionless amplitude of the GW background generated by black hole formation as a function of the SFR density, and related to the 'first light' epoch.

It is worth mentioning that the formulation used here is similar to that used by Ferrari et al. (1999a), but instead of using an average energy flux taken from Stark & Piran (1986), who simulated the axisymmetric collapse of a rotating polytropic star to a black hole, we use equation (10) to obtain the energy flux, which takes into account the most relevant quasi-normal modes of a rotating black hole and represents a kind of average over the rotational parameter (see de Araujo et al. 2000). Both formulations present similar results, since in the end the most important contributions to the energy flux come from the quasi-normal modes of the black holes formed, which account for most of the gravitational radiation produced during the collapse process.

The SFR density, however, for the formation of the first stars is unknown. Star formation in other media could in principle give us some information on how things occurred in the 'first light' epoch, but unfortunately as one can see in what follows there are no compelling arguments in this direction.

The formation of a bound cluster of stars requires a star formation efficiency of ~ 50 per cent when the cloud disruption is sudden and ~ 20 per cent when cloud disruption takes place on a longer time-scale (Margulis & Lada 1983; Mathieu 1983; Ciardi et al. 2000). We define 'star formation efficiency' as the fraction of gas of a cloud that is converted into stars [this definition is similar to that used by Ciardi et al. (2000), among others].

On the other hand, Pandey, Paliwal & Mahra (1990) have investigated the influence of the IMF on the star formation efficiency, for clouds of different masses, and have concluded that the efficiency decreases if massive stars (the most destructive ones) are formed earlier. Some studies have also analysed the molecular gas properties and star formation in nearby nuclear starburst galaxies (see, e.g., Planesas, Colina & Perez-Olea 1997) indicating the existence of giant molecular clouds with masses $\sim 10^8 - 10^9{\rm M}_\odot$, in which star formation process occurs in a short time ($< 3\times 10^7$ yr) with efficiency of conversion of gas into stars $\lesssim 10$ per cent. All these studies show the large uncertainties in the star formation efficiency.

Probably, the best way to infer the SFR density is to relate it to studies concerning the reionization of the Universe. In the introduction of the present paper we argued that there are compelling arguments in favor of a reionization phase of the Universe, which probably occurred at redshifts in the range $6 < z < 30$ (Tegmark, Silk & Blanchard 1994; Venkatesan 2000). There are at least two compelling reasons for reionization through Population III stars to be considered. First, the Population III stars are expected to form at $z \gtrsim 10$, being capable of ionizing hydrogen. Secondly, the first stars create heavy elements, and can account for the metalicity of $\sim 10^{-2}Z_\odot$ found in Ly $-\ \alpha$ forest clouds (Venkatesan 2000). It is found that the amount of baryons necessary to participate in early star formation, to account for the reionization, would amount to a small fraction, $f_\star$, of all baryons of the Universe (see, e.g., Venkatesan 2000; Loeb & Barkana 2001).

The above discussion suggests that we can write the SFR density as follows:

$$\dot\rho_\star \equiv \frac{d\rho_\star}{dt} = \frac{d}{dt}[\Omega_\star\ \rho_{\rm c}\ (1+z)^3], \tag{21}$$

where the term in brackets represents the stellar mass density at redshift z, with $\rho_{\rm c}$ the present critical density and $\Omega_\star$ the stellar density parameter. The latter can be written as a fraction of the baryonic density parameter, namely, $\Omega_\star = f_\star\Omega_{\rm B}$.

Another relevant physical quantity associated with the GW background, produced by the first stars, is the closure energy density per logarithmic frequency span, which is given by

$$\Omega_{\rm GW} = \frac{1}{\rho_{\rm c}} \frac{d\rho_{\rm GW}}{d \log \nu_{\rm obs}}. \tag{22}$$

The above equation can be rewritten as

$$\Omega_{\rm GW} = \frac{\nu_{\rm obs}}{c^3 \rho_{\rm c}} F_\nu = \frac{4\pi^2}{3H_0^2} \nu_{\rm obs}^2 h_{\rm BG}^2. \tag{23}$$

In the next section we present the numerical results and discussions, which come mainly from equation (14). Looking at this equation one notes that, to integrate it, one needs to choose the IMF, and to set values for the following parameters: $z_{\rm ci}$, $z_{\rm cf}$, α, $\varepsilon_{\rm GW}$, $f_\star$, H_0, $\Omega_{\rm B}$, $\Omega_{\rm M}$ and Ω_Λ.

4 NUMERICAL RESULTS AND DISCUSSIONS

Based on models of structure formation, the first objects that collapsed should have had masses around $10^6 {\rm M}_\odot$ (see, e.g., Tegmark et al. 1997, among others), and objects with higher masses should have collapsed subsequently. Evidently, as the density fluctuations could have had peaks higher than 1σ values, clouds could have collapsed earlier. As a result, there were clouds with different masses collapsing around the redshift of collapse of $10^6 {\rm M}_\odot$.

We have considered that the first stellar formation is related to the collapse of Population III objects. To evaluate the GW background produced by the formation of the Population III black holes, it is necessary to know the redshifts at which they began and finished being formed. This is a very hard question to answer, since it involves knowledge of the role of the negative and positive feedbacks of star formation, which are regulated by cooling and injection of energy processes.

Should the stochastic GW background studied here be significantly produced and detected at a reasonable confidence level, the present study can be used to obtain the redshift range where the Population III black holes were formed, independently of any CDM modelling. In Fig. 1 an example is given of how one could get $z_{\rm ci}$ and $z_{\rm cf}$ from the curve $h_{\rm BG}$ versus $\nu_{\rm obs}$. Knowing the frequency band $\nu_{\rm min} - \nu_{\rm max}$ and using equation (12), one obtains $z_{\rm ci}$ and $z_{\rm cf}$ (see Fig. 1), which are, therefore observable. Note that we have assumed as did Ferrari et al. (1999a,b) that α is a constant ($\alpha = 0.1$).

Is is worth noting that α may depend sensitively on the metallicity: the lower the value of Z, the higher are the remnant masses and the less ejected material there is relative to $Z_\odot$ stars. More realistically there would be a dependence of α on the progenitor mass. On the other hand, the value $\alpha = 0.1$ adopted can be considered as a mean value for the progenitor masses studied here. If α is not very well determined, this would mean that the observed frequency band does not uniquely fix the redshift band where the black holes are formed.

In the introduction of the present paper we mention that different studies related to the reionization of the Universe set this epoch as being somewhere in the range $6 < z < 30$. It is also clear that the reionization process is not instantaneous. Stars start forming at different redshifts,

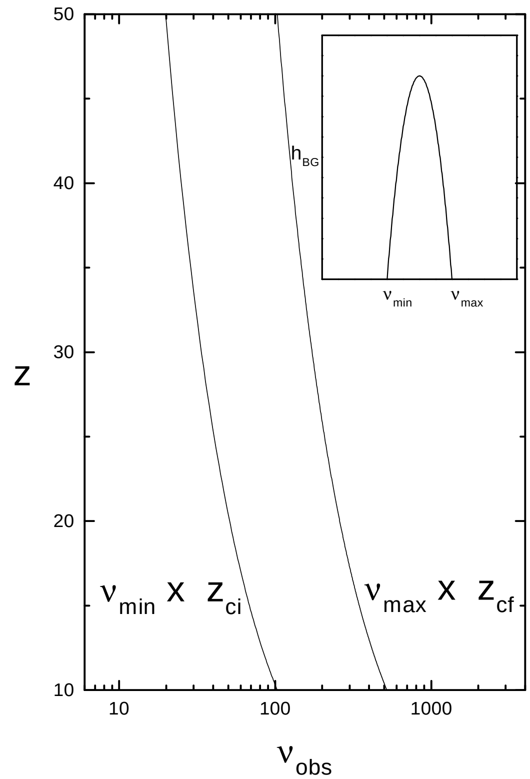

Figure 1. Example of how one could obtain from $h_{\rm BG}$ versus $\nu_{\rm obs}$ the initial (final) redshift $z_{\rm ci}$ ($z_{\rm cf}$) of the GW emission period. We have adopted $\alpha = 0.1$.

creating ionized bubbles (Strömgren spheres) around themselves, which expand into the intergalactic medium (IGM), at a rate dictated by the source luminosity and the background IGM density (Loeb & Barkana 2001). The reionization is complete when the bubbles overlap to fill the entire Universe. Thus the epoch of reionization is not the epoch of star formation. There is a non-negligible time-span between them. Here, we have chosen different formation epochs to see their influence on the putative GW background and also to see if it could be detected by the forthcoming GW antennas.

The first relevant quantity appearing in the equation for the GW background is the IMF. As discussed in the previous section, we have adopted the standard IMF ($x = 1.35$), as our fiducial case, and have also studied two other cases to assess the role of the IMF variations on $h_{\rm BG}$.

The second relevant parameter is $\varepsilon_{\rm GW}$, the efficiency of production of GWs, whose distribution function is unknown. Thus, we have parameterized our results in terms of its maximum value, namely, $\varepsilon_{\rm GW_{max}} = 7 \times 10^{-4}$, which is obtained from studies by Stark & Piran (1986) who simulated the axisymmetric collapse of a rotating star to a black hole. We will see below that, if $\varepsilon_{\rm GW}$ is a very tiny fraction of the maximum value, the detection of the GW background whose existence we propose, is very improbable, even for advanced antennas.

To calculate $h_{\rm BG}$ we still need to know $\Omega_\star$, which has a key role in the definition of the SFR density. As discussed in the previous section studies related to the reionization of the Universe can shed some light on $\Omega_\star$. From different studies one can conclude that a few percent, maybe up to ~ 10 per cent, of the baryons must be condensed into stars in order for the reionization of the Universe to take place. Here we have set the value of $\Omega_\star$ in such a way that it amounts to 1 per cent of all baryons (our fiducial value). In the next section we discuss the detectability of the GW background, and we then parameterize our results in terms of $f_\star = \Omega_\star/\Omega_{\rm B}$.

Looking at equation (14) one could think it would depend critically on the cosmological parameters: H_0, $\Omega_{\rm B}$, $\Omega_{\rm DM}$, Ω_Λ. However, our results show that, given the redshifts involved in our calculations, $h_{\rm BG}$ depends only on H_0 and $\Omega_{\rm B}$; the latter dependence occurs because this parameter appears in the SFR density. The quantity $h_{100}^2\Omega_{\rm B} = 0.019 \pm 0.0024$ (where h_{100} is the Hubble parameter given in terms of 100 km s^{-1} Mpc^{-1}) is obtained from big bang nucleosynthesis studies (see, e.g., Burles et al. 1999).

In Table 1 we present the redshift band, $z_{\rm ci}$ and $z_{\rm cf}$ for the models studied and the corresponding GW frequency bands. For the cosmological parameters we have adopted $h_{100} = 0.65$, $\Omega_{\rm M} = 0.3$, $\Omega_{\rm B} = 0.045$ and $\Omega_\Lambda = 0.7$. Keep in mind that our results are sensitive to the combination $h_{100}^2\Omega_{\rm B}$. We have also adopted $\alpha = 0.1$, $f_\star = 0.01$ and the standard IMF.

Note that no structure formation model has been used to find the black hole formation epoch. Instead we have simply chosen the values of z to see whether it is possible to obtain detectable GW signals. In the next section it will be seen that, unless $\varepsilon_{\rm GW}$ is negligible, the GW background that we propose here can be detected. Our choices, however, can be understood as follows. The greater the redshift formation, the more power the masses related to the Population III objects have. Thus, of our models A to D, our model D (A) has more (less) power when compared to the others. Models E, F and G would mean a more extended star formation epoch, which means that the feedback processes of star formation are such that they allow a more extended star formation epoch when compared to models B, C and D, respectively.

Concerning the reionization epoch, as already mentioned, it occurred at lower redshifts as compared to the first star formation redshifts. Loeb & Barkana (2001) found, for example, that if the stars were formed at $z \simeq 10 - 30$, with standard IMF, they could have reionized the Universe at redshift $z \sim 6$. Our models A, B and E, for example, could account for such a reionization redshift.

In addition, we also consider a model with $z_{\rm cf} = 5$ (see model H of Table 1) to verify if a final epoch of star formation close to this redshift could produce a detectable signal for the VIRGO and LIGO experiments.

If the process of structure formation of the Universe and the consequent star formation were well known, one could obtain the redshift formation epoch of the first stars. On the other hand, as discussed below, if the GW background really exists and is detected, one can obtain information about the formation epoch of the first stars.

A relevant question is whether the background we study here is continuous or not. The duty cycle indicates if the collective effect of the bursts of GWs generated during the collapse of a progenitor star generates a continuous background. The duty cycle is defined as follows:

Table 1. The redshifts of collapse for our models and the corresponding GW frequency bands. The cosmological parameter $h_{100}^2\Omega_{\rm B} = 0.019$ (see the text), $\alpha = 0.1$, $f_\star = 0.01$ (our fiducial value) and the standard IMF are adopted.

Model	$z_{\rm ci}$	$z_{\rm cf}$	$\Delta\nu$ (Hz)
A	20	10	50-470
B	30	20	34-250
C	40	30	25-170
D	50	40	20-130
E	30	10	34-470
F	40	10	25-470
G	50	10	20-470
H	15	5	65-870

$$DC = \int_{z_{\rm cf}}^{z_{\rm ci}} dR_{\rm BH}\Delta\bar{\tau}_{\rm GW}(1+z), \tag{24}$$

where $\Delta\bar{\tau}_{\rm GW}$ is the average time duration of single bursts at the emission, which is inversely proportional to the frequency of the lowest quasi-normal mode of the rotating black holes (see, e.g., Ferrari et al. 1999a), which amounts to $\sim 1\,$ms for the mass range of the black holes considered here.

Since the star formation rate could be high, a significant amount of GWs could be produced. We also note that, independently of the primordial cloud mass and of redshift of collapse, star formation occurring at high redshift could produce high duty cycle values, which lead us to conclude that the stochastic GW background could be continuous. For all the models studied here the duty cycle is $\gg 1$.

The amplitude $h_{\rm BG}$ of the GW background in terms of $\varepsilon_{\rm GW}$ (the efficiency of generation of GWs), in the frequency band $\nu_{\rm min} - \nu_{\rm max}$ is shown in Fig. 2 for the models A, E and F of Table 1. In the next section we discuss the detectability of such a putative background.

Note that, the earlier that star formation occurs, the greater is the GW amplitude $h_{\rm BG}$. This can be explained as follows. For a given individual source, the higher the redshift, the lower is the amplitude of the GWs generated. On the other hand, the higher the redshift of star formation, the greater is the SFR density. As a result there is a more significant overlapping of bursts of GWs at higher redshifts.

We find, for example, that the formation of Population III of black holes, in model D, could generate a stochastic GW background with amplitude $h_{\rm BG} \simeq (0.8 - 2) \times 10^{-24}$ and a corresponding closure density of $\Omega_{\rm GW} \simeq (0.7 - 1.4) \times 10^{-8}$, in the frequency band $\nu_{\rm obs} \simeq 20 - 130\,$Hz (assuming an efficiency of generation $\varepsilon_{\rm GW} \simeq 7 \times 10^{-4}$, the maximum one).

It could be argued that the formation of stars could not be restricted to the epoch of the collapse of the first Population III objects. A new surge of star formation associated with the collapse of Population III objects of $M > 10^6{\rm M}_\odot$ at lower redshifts could occur. The existence of this new surge of star formation would depend on the role of the negative and positive feedbacks, which are regulated by cooling

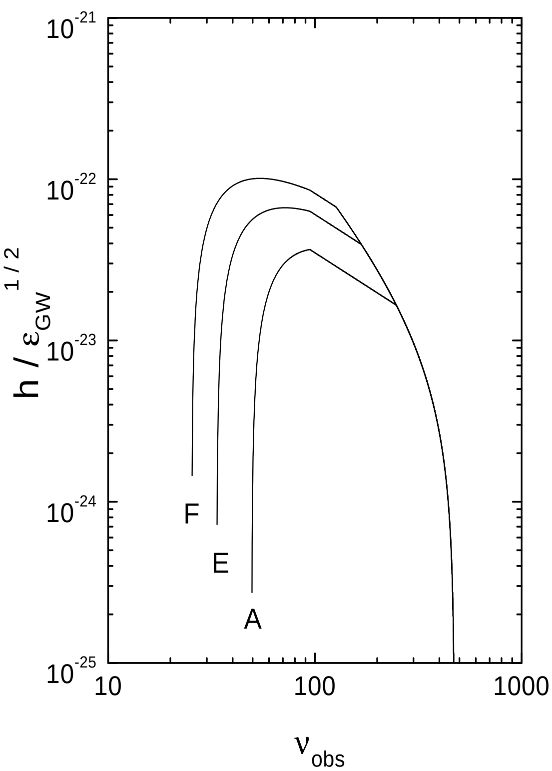

Figure 2. The background amplitude of the GWs as a function of $\nu_{\rm obs}$ and the efficiency of GW generation $\varepsilon_{\rm GW}$ for models A, E and F of Table 1

and injection of energy processes of the previous star formation surge. If another surge of star formation took place, then another GW background could be generated, and a partial superposition with the background previously generated could also occur. As a result for some frequency bands the GW amplitude could be enhanced.

Another possibility would be a star formation process taking place during the time of collapse of the first Population III objects and continuing during the collapse of objects of higher masses. In such a case the star formation would occur for a large redshift span, and as a result the frequency band, the amplitude and the closure energy density of GWs could be larger. As before, the role of the negative and positive feedbacks of the star formation would be the key point. In the models E, F and G we have considered such a possibility (see also Fig. 2).

Certainly, the GW background produced depends on the star formation history. A different star formation history would produce different results for both the values of $h_{\rm BG}$ and frequency bands. However, it is hard to avoid the conclusion that, if the first stars had been formed at high redshift, a significant amount of GWs would have been produced as well.

To assess the role of possible IMF variations we have considered, besides the standard IMF, two others, namely, with $x = 0.3$ and $x = 1.85$, which yield ten times and one-tenth of the mass fraction of black holes of the standard IMF, respectively. For the model A, our calculations show that for $x = 0.3$ ($x = 1.85$) the maximum $h_{\rm BG}$ is a factor $\simeq 3$ ($\simeq 4$) greater (smaller) than case with standard IMF ($x = 1.35$). In the next section we consider the role of the IMF variations on detectability of the background of the GW background that we propose exists.

5 DETECTABILITY OF THE BACKGROUND OF GRAVITATIONAL WAVES

The background predicted in the present study cannot be detected by single forthcoming interferometric detectors, such as VIRGO and LIGO (even by advanced ones). However, it is possible to correlate the signal of two or more detectors to detect the background that we propose exists. Michelson (1987) was the first to show that this kind of signal can, in principle, be detected by correlating the outputs of two different detectors. However, the main requirement that must be fulfilled is that they must have independent noise. This study was improved by Christensen (1992) and by Flanagan (1993). The reader should also refer to the papers by Allen (1997) and Allen & Romano (1999) who also deal in detail with such an issue.

To assess the detectability of a GW signal, one must evaluate the signal-to-noise ratio (S/N), which for a pair of interferometers is given by (see, e.g., Flanagan 1993; Allen 1997)

$$({\rm S/N})^2 = \left[\left(\frac{9H_0^4}{50\pi^4} \right) T \int_0^\infty d\nu \frac{\gamma^2(\nu)\Omega_{GW}^2(\nu)}{\nu^6 S_h^{(1)}(\nu) S_h^{(2)}(\nu)} \right] \qquad (25)$$

where $S_h^{(i)}$ is the spectral noise density, T is the integration time and $\gamma(\nu)$ is the overlap reduction function, which depends on the relative positions and orientations of the two interferometers. For the $\gamma(\nu)$ function we refer the reader to Flanagan (1993), who was the first to calculate a closed form for the LIGO observatories. Flanagan (1993; see also Allen 1997) showed that the best window for detecting a signal is $0 < \nu < 64$ Hz, where the overlap reduction function has the greatest magnitude.

Here we consider, in particular, the LIGO interferometers. Their spectral noise densities have been taken from a paper by Owen et al. (1998)- who in turn obtained them from Thorne, by means of private communication.

In Table 2 we present the S/N for one year of observation with $\alpha = 0.1$, $\Omega_{\rm B} h_{100}^2 = 0.019$, $f_\star = 0.01$ and $\varepsilon_{{\rm GW}_{\rm max}} = 7 \times 10^{-4}$ for the models of Table 1, for the three LIGO interferometer configurations.

Note that for the 'initial' LIGO (LIGO I) there is no hope of detecting the GW background we propose here, even for ideal orientation and locations of the interferometers, i.e., $|\gamma(\nu)| = 1$. For the 'enhanced' LIGO (LIGO II) there is some possibility of detecting the background, since $\rm S/N > 1$, if $\varepsilon_{\rm GW}$ is around the maximum value. Even if the LIGO II interferometers cannot detect such a background, it will be possible to constrain the efficiency of GW production.

The prospect for the detection with the 'advanced' LIGO (LIGO III) interferometers is much more optimistic, since the S/N for almost all models is significantly greater than unity. Only if the value of $\varepsilon_{\rm GW}$ were significantly lower

Table 2. For the models of Table 1 we present the S/N for pairs of LIGO I, II and III ('first', 'enhanced' and 'advanced', respectively) observatories for one year of observation. Note that an efficiency of generation $\varepsilon_{\rm GW_{max}} = 7 \times 10^{-4}$ is assumed.

Model		S/N	
	LIGO I	LIGO II	LIGO III
A	8.3×10^{-3}	1.6	6.6
B	8.5×10^{-3}	2.3	26
C	8.7×10^{-3}	2.7	47
D	8.1×10^{-3}	2.5	51
E	2.7×10^{-3}	5.7	37
F	5.0×10^{-3}	12	120
G	7.7×10^{-2}	21	260
H	4.6×10^{-3}	0.5	1.7

than the maximum value would the detection not be possible. In fact, the S/N is critically dependent on this parameter, whose distribution function is unknown.

Note, for example, that it is possible to detect a GW background with the 'advanced' LIGO, even for star formation for which $z_{\rm cf} \sim 5$ (model H), if $\varepsilon_{\rm GW}$ is around the maximum value.

Let us now look at how the variations of the parameters modify our results. First of all, note that the larger the star formation redshift band, the greater is the S/N. Secondly, the earlier the star formation, the greater is the S/N. It is worth recalling that, if one can obtain the curve of $h_{\rm BG}$ versus $\nu_{\rm obs}$ and if the value of α is known, one can find the redshift of star formation.

The S/N is also sensitive to variations of α. The larger α, the lower are the GW frequencies and the higher is $h_{\rm BG}$, and since the best window for detection is around $0 < \nu < 64$Hz, the S/N is higher.

Even if α is not known beforehand, it is possible to impose a constraint on its values, and also on the redshift star formation epoch. For example, if one found from GW observations that the GW frequency band were $40 - 200$ Hz, one would obtain (using Equation 11) that $\alpha \simeq 0.1 - 0.4$ and $z_{\rm f} \simeq 5 - 50$. On the other hand, if one knew that the star formation redshift band were $z_f \simeq 10 - 30$, through some model of structure formation or whatever observational data, and the GW frequency band were known, say $40 - 200$ Hz, one would obtain that $\alpha \simeq 0.1$.

It would be interesting to perform a study considering α as a function of the progenitor mass, which would result in a more realistic model. There are some studies in the literature considering how the remnant mass depends on the progenitor (see, e.g., Fryer & Kalogera 2001), but we will not consider such an issue here.

To assess the role of the variations in the IMF, we modify its exponent x. For $x = 0.3$ ($x = 1.85$), the S/N is ~ 10 (~ 0.1) times the S/N of the standard IMF. As expected for an IMF biased toward high- (low-) mass stars, where one has a greater (lower) amount of black holes, the S/N is greater (lower).

Note that the S/N for a given formation epoch, IMF and α, and for one year of observation, still presents a dependence on $\Omega_{\rm B} h^2$, $f_\star$ and $\varepsilon_{\rm GW}$, namely

$$\mathrm{S/N} \propto \Omega_{\rm B} h^2 f_\star \, \varepsilon_{\rm GW}. \tag{26}$$

The value of $\Omega_{\rm B} h^2$ is well constrained by primordial nucleosynthesis studies. For $f_\star$ we have adopted a value of 0.01, which is a very conservative choice. Note that the value for this parameter can be obtained from studies concerning the reionization of the Universe, which is very difficult to model, but there is some agreement in the literature (see, e.g, Gnedin 2000; Venkatesan 2000; Loeb & Barkana 2001) among the different models of the reionization of the Universe which lead us to conclude that $f_\star$ could range from a few up to 15 per cent.

For $\varepsilon_{\rm GW}$, the situation is more complicated since its distribution function is unknown. We have adopted here the maximum value as a reference, but if its actual value is much less than this value the S/N could be lower than unity for all the models studied here, even for a LIGO III pair. Let us think of what occurs with other compact objects, namely, the neutron stars, to see if we can learn something from them. Hot and rapidly rotating neutron stars can lose angular momentum to gravitational radiation via the so-called r-mode instability (Anderson 1998). This could explain why all known young neutron stars are relatively slow rotators. The black holes could have had a similar history, i.e., they could have been formed rapidly rotating and lost angular momentum to gravitation radiation via their quasi-normal modes. If this was the case, the value of $\varepsilon_{\rm GW}$ could be near the maximum one, or in the worst case it could have a value to produce $\mathrm{S/N} > 1$ at least for a LIGO III pair.

In order to assess the values of $f_\star$ and $\varepsilon_{\rm GW}$ that yield $\mathrm{S/N} > 1$, for a given formation epoch, IMF, α and $\Omega_{\rm B} h^2$, we present in Fig. 3 the regions in the $(f_\star, \varepsilon_{\rm GW})$ plane where S/N could be greater than unity for a pair of LIGO III interferometers. Note that unless $\varepsilon_{\rm GW}$ is very small, S/N can be significantly greater than unity, indicating that the background could in principle be detected in the near future.

A relevant issue is whether there are other GW backgrounds that could be confused with that of the present study. Relic GWs generated in the very early Universe can in principle present a signal in the LIGO bandwidth. The ordinary inflationary models, however, predict $\Omega_{\rm GW} \sim 10^{-15}$ (see, e.g., Schultz 1999; Giovannini 2000; Maggiore 2000a,b). This is much less than our models predict, and therefore undetectable even with a pair of LIGO III interferometers. Other models, such as string cosmologies, provide different predictions (see, e.g., Schutz 1999) with values of $\Omega_{\rm GW}$ that could be much greater than our studies predict, which could render the background studied here undetectable. Other GW backgrounds exist in the bandwidth of LIGO, which could have been produced at $0 < z < 5$, namely: (a) a cosmological population of core-collapse supernovae (Ferrari et al. 1999a); (b) a population of young rapidly rotating neutron stars (Owen et al. 1998, Ferrari et al. 1999b); and (c) double neutron star binaries (Schneider et al. 2000). The first two backgrounds, however, have energy density shifted for higher GW frequencies when compared to our predictions. Moreover, the S/Ns of these backgrounds are less than unity, even for a pair of LIGO III. The last one has a frequency band ranging from $\sim 10^{-5}$ Hz up to $\sim 10^{2}$ Hz, and therefore there is a partial overlap with the background of our study. The GW amplitudes, however, would be comparable only if $\varepsilon_{\rm GW}/\varepsilon_{\rm GW_{max}} << 1$.

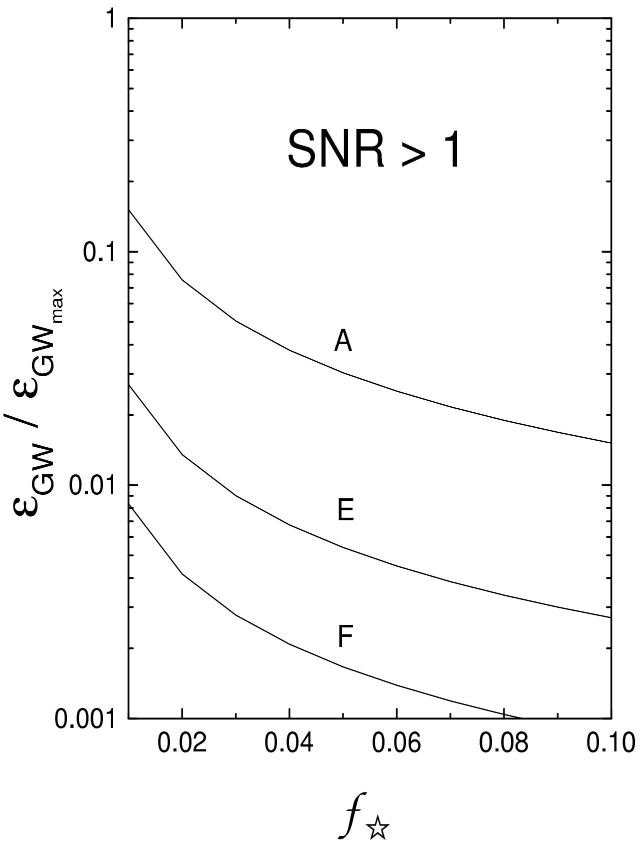

Figure 3. Relative efficiency of GW generation, $\varepsilon_{\rm GW}/\varepsilon_{\rm GW_{max}}$, as a function of the fraction of baryons participating in the early star formation, $f_\star$, for models A, E and F of Table 1 for a pair of LIGO III interferometers. The curves represent where the S/N = 1; above them S/N > 1. As in Table 2, $\varepsilon_{\rm GW_{max}} = 7 \times 10^{-4}$ is assumed.

6 CONCLUSIONS

We present here a study concerning the generation of GWs produced from a cosmological population of black holes. These objects are formed as a consequence of the collapse of pre-galactic objects (Population III objects) that form the first generation of stars at high redshift. Our results show that different structure formation models which predict the formation of the first objects at $z > 10$ could, in principle, predict the formation of pre-galactic black holes and a significant stochastic GW background associated with them. Our results lead us to conclude that star formation occurring at high redshifts could have large duty cycles and so the stochastic GW background generated is continuous.

We consider that stars are formed following a Salpeter IMF and having masses in the range $0.1 - 125 {\rm M}_\odot$. Certainly, the results presented here are dependent on this particular choice. A steeper IMF would modify the number of high-mass stars, modifying the peak of $h_{\rm BG}$ and the frequency band of the GWs. For an IMF with $x = 0.30$ ($x = 1.85$) the IMF is biased toward high- (low-) mass stars, as a result the S/N is ∼10 (∼0.1) times the S/N predicted with the use of a standard IMF. It would be of interest, however, to have a look in detail at studies of the metallicity of high-z Ly α clouds to see if it is possible to constrain the Population III IMF.

If we consider $\varepsilon_{\rm GW} \simeq 7.0 \times 10^{-4}$ (see, e.g., Stark & Piran 1986) then we obtain $h_{\rm BG} \simeq (0.8 - 2) \times 10^{-24}$ and $\Omega_{\rm GW} \simeq (0.7 - 1.4) \times 10^{-8}$ at $\nu_{\rm obs} \simeq 20 - 130$ Hz for the model D. Thus, this GW background produced as a consequence of the formation of the first stars in the Universe is capable of being detected by a pair of 'advanced' LIGO interferometers.

As seen, with reasonable parameters, our results show that a significant amount of GWs is produced related to the the Population III black hole formation at high redshift, and can in principle be detected by a pair of LIGO II (or most probably by a pair of LIGO III) interferometers. However, a relevant question should be considered: What astrophysical information can one obtain from whether or not such a putative background is detected?

First, let us consider a non-detection of the GW background. The critical parameter to be constrained here is $\varepsilon_{\rm GW}$. A non-detection would mean that the efficiency of GWs during the formation of black holes is not high enough. Another possibility is that the first generation of stars is such that the black holes formed had masses $> 100 M_\odot$, and should they form at $z > 10$ the GW frequency band would be out of the LIGO frequency band.

Secondly, a detection of the background with a significant S/N would permit us to obtain the curve $h_{\rm BG}$ versus $\nu_{\rm obs}$. From it, as discussed above, one can constrain α and the redshift formation epoch; and for a given IMF and $\Omega_{\rm B} h^2$, one can also constrain the values of $f_\star$ and $\varepsilon_{\rm GW}$. On the other hand, using the curve $h_{\rm BG}$ versus $\nu_{\rm obs}$ and in addition other astrophysical data, say CBR data, models of structure formation and reionization of Universe, a constraint on $\varepsilon_{\rm GW}$ can also be imposed.

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